Integrated Mathematics III

Standard 1

Algebra and Functions

CORE STANDARDS

Quadratic Equations and Functions

Graphing Quadratic Functions

Graph quadratic functions. Determine how the graph of a parabola changes if a, b, and c are changed in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola given sufficient information.

[Standard Indicators: IM3.1.4, IM3.1.6]

Solving Quadratic Equations

Solve quadratic equations in the complex number system. Solve problems that can be modeled using quadratic equations and functions.

[Standard Indicators: IM3.1.4, IM3.1.7]

CORE STANDARDS

Complex Numbers

Add, subtract, multiply and divide complex numbers.

[Standard Indicator: IM3.1.7]

CORE STANDARDS

Polynomial Equations and Functions

Writing Polynomial Equations

Perform arithmetic operations, including long division, on polynomials. Find a polynomial when given its roots and use the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression to solve problems.

[Standard Indicators: 1M3.1.10, IM3.1.14]

Solving Polynomial Equations

Solve polynomial equations by factoring. Solve problems that can be modeled using polynomial equations.

[Standard Indicators: IM3.1.11, IM3.1.13]

CORE STANDARDS

Rational Functions

Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative

exponents in the denominator. Solve problems that can be modeled using equations involving rational functions.

[Standard Indicators: IM3.1.16]

CORE STANDARDS

Exponential and Logarithmic Equations

Use laws of exponents to derive laws of logarithms. Use laws of logarithms to solve problems. Solve exponential and logarithmic equations. Solve problems that can be modeled using equations involving exponents and logarithms.

[Standard Indicators: IM3.1.22, IM3.1.23, IM3.1.24]

CORE STANDARDS

Sequences and Series

Find specific terms of arithmetic and geometric sequences. Find partial sums of arithmetic and geometric series. Solve problems that can be modeled using arithmetic and geometric series.

[Standard Indicators: IM3.1.25, IM3.1.26, IM3.1.27]

IM3.1.1 Find the zeros, domain and range of a function.

Example: $f(x) = x^3 - 3x^2 - x + 3$. What is the domain and range of this function?

IM3.1.2 Define, add, subtract, multiply and divide complex numbers. Represent complex numbers and the addition, subtraction and absolute value of complex numbers in the complex plane.

Example: Let $z = 7 - 4i$ and w = $10 + 6i$. Graph z, w and $z + w$. Prove that the number 0, z, w and $z + w$ are the vertices of a parallelogram on the complex plane.

IM3.1.3 Solve quadratic equations in the complex number system.

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

IM3.1.4 Analyze, describe and sketch graphs of quadratic functions and include the lines of symmetry.

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

IM3.1.5 Determine how the graph of a parabola changes if a, b and c changes in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola when given sufficient information.

Example: Write the equation of the parabola with vertex (3, 6), y-intercept 2 in vertex form.

IM3.1.6 Solve problems that can be modeled using quadratic equations and functions, interpret the solutions, and determine whether the solutions are reasonable.

Examples:

- Write the equation of the parabola with vertex (3, 6) and y-intercept 2 in vertex form.
- Describe similarities and differences in the graphs of $y = 2x^2$ and $y = 2(x - 1)^2 + 3$ without first graphing the equations.

IM3.1.7 Analyze, describe and sketch graphs of polynomial functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Determine by inspection the end behavior of the graph of the function $f(x) = -2x^3 + x^2 + 4x - 5$.

IM3.1.8 Use the binomial theorem to expand binomial expressions raised to positive integer powers.

Example: Expand $(x + 2)^4$.

IM3.1.9 Perform arithmetic operations, including long division and division with remainders, on polynomials by others of equal or lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

IM3.1.10 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 + 27 = 0$ by factoring.

IM3.1.11 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

IM3.1.12 Solve problems that can be represented or modeled using polynomial equations, interpret the solutions and determine whether the solutions are reasonable.

Example: You want to make an open-top box with a volume of 500 cubic inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Then use your results to give a formula for the volume of the box.

IM3.1.13 Find a polynomial function of lowest degree with real coefficients when given its roots. Solve problems by using the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph and factors of a polynomial expression.

Example: Write an equation that has solutions $x = 2$, $x = 5i$ and $x = -5i$.

IM3.1.14 Analyze, describe and sketch graphs of rational functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Find the equations of the horizontal and vertical asymptotes of the function

$$f(x) = \frac{x+1}{x+5}$$

IM3.1.15 Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator.

Example: Simplify $\frac{x^2-4}{x^5} \div \frac{x^3-8}{x^8}$.

IM3.1.16 Understand the properties of rational exponents and use the properties to simplify, multiply, divide and find powers of expressions containing negative and fractional exponents. Relate expressions containing rational exponents to the corresponding radical expressions.

Example: Write the expression $\left(x^{\frac{1}{2}} y^{\frac{-2}{3}} \right)^6$ in simplest form. Assume all variables are positive.

IM3.1.17 Analyze, describe and sketch graphs of square root and cube root functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Graph the function $y = \sqrt{x} + 7$ and find the domain and range.

IM3.1.18 Solve equations that contain radical expressions and identify extraneous roots when they occur.

Example: Solve the equation $x = \sqrt{x} + 2$.

IM3.1.19 Solve problems that can be modeled using equations involving rational and radical functions, including problems of direct and inverse variation. Interpret the solutions and determine whether the solutions are reasonable.

Example: Two students working independently can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together at the same rate as when doing the job alone?

IM3.1.20 Analyze, describe and sketch graphs of exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

IM3.1.21 Know that the inverse of an exponential function is a logarithm. Use laws of exponents to derive laws of logarithms. Use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve problems.

Example: If you know that log(2) = *a* and log(3) = *b*, find log(36) in terms of *a* and *b*.

IM3.1.22 Solve exponential and logarithmic equations.

Example: Solve the equation $\log_2 x = 5$.

IM3.1.23 Solve problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable. Use technology as appropriate.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million and 400 million. What do you notice about these time periods?

IM3.1.24 Write the recursive formula for arithmetic and geometric sequences and find specified terms of arithmetic and geometric sequences.

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15 ….

IM3.1.25 Write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions.

Example: Write the formula for the general term of the geometric sequence 2, 6, 18, 54, 162....

IM3.1.26 Find partial sums of arithmetic and geometric series.

Example: In the last example (IM3.1.25), find the sum of the first 10 terms.

IM3.1.27 Solve problems involving applications that can be modeled using sequences and finite arithmetic and geometric series. Interpret the solutions and determine whether the solutions are reasonable using spreadsheets as appropriate.

Example: A restaurant has square tables that seat four people. When two tables are placed together, six people can be seated. If 20 square tables are placed together to form one long table, how many people can be seated?

Standard 2

Geometry and Measurement

CORE STANDARD

Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry. Use these measures to show whether shapes are similar or congruent and whether line segments are parallel or perpendicular.

[Standard Indicators: IM3.2.1, IM3.2.5]

CORE STANDARD

Triangles

Develop simple geometric proofs involving triangles.

[Standard Indicator: IM3.2.4]

CORE STANDARD

Circles

Find the equation of a circle in the coordinate plane. Determine how the graph of a circle changes if a, b, and r are changed in the equation $(x - a)^2 + (y - b)^2 = r^2$.

[Standard Indicator: IM3.2.5]

CORE STANDARD

Solids

Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas.

[Standard Indicator: IM3.2.6]

IM3.2.1 Represent geometric objects and figures algebraically using coordinates, use algebra to solve geometric problems, and develop simple coordinate proofs involving geometric objects in the coordinate plane.

Example: Draw a triangle with vertices at (1, 3), (2, 5) and (6, 1). Draw another triangle with vertices (-3, -1), (-2, 1) and (2, -3). Are these triangles the same shape and size? Justify your answer.

IM3.2.2 Construct triangles congruent to given triangles. Explain and justify the process used.

Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

IM3.2.3 Develop simple geometric proofs involving triangles and provide reasons for each statement of the proof.

Example: Prove the following:

- The sum of the angles in a triangle is 180°.
- The line joining the midpoint of two sides of a triangle is parallel to, and half the length of, the third side.
- The perpendicular bisectors of the sides of a triangle meet at a point that is the center of the circle and that contains the vertices of the triangle.

IM3.2.4 Find the equation of a circle in the coordinate plane in terms of its center and radius and determine how the graph of a circle changes if a, b and r are changed in the equation $(x - a)^2 + (y - b)^2 = r^2$.

Example: Find the equation of the circle with radius 10 and center (6, -3).

IM3.2.5 Find and use measures of sides, volumes and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres. Relate these measures to each other using formulas.

Example: A marble is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the marble?

IM3.2.6 Visualize solids and surfaces in three-dimensional space when given two-dimensional representations, and create two-dimensional representations for the surfaces of three-dimensional objects.

Example: Make a net for a tetrahedron out of poster board and fold it to make the tetrahedron.

IM3.2.7 Describe the structure of and relationships within an axiomatic system (e.g., undefined terms, definitions, axioms and postulates, methods of reasoning, and theorems).

Example: Do you prove axioms from theorems or theorems from axioms?

IM3.2.8 Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true. Illustrate its counterparts in other geometries.

Example: Describe and illustrate at least one non-Euclidean geometry postulate.

Standard 3
Data Analysis and Statistics

IM3.3.1 Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.

Example: About 30 percent of the students at a school are on the honor roll. If you took a random sample of 30 students, what range of students would likely be on the honor roll?

IM3.3.2 Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis and the validity of conclusions. Interpret confidence levels and "margin of error."

Example: In a random poll of 1,025 women, it was found that 47 percent of the women polled said they do not get enough time for themselves. The poll announced a margin of error of ± 3 percent points for 95 percent confidence in its conclusion. Explain to someone who knows no statistics why it cannot be said that 47 percent of all adult women do not get enough time for themselves. Then explain what "95 percent confidence" means.

IM3.3.3 Compare the differences among surveys, experiments and observational studies and recognize which types of inferences can legitimately be drawn from each.

Example: Gather data to answer the question: Which area of the country has the highest high school dropout rate? Display your dropout data in various forms.

IM3.3.4 Compute basic statistics (mean, median, weighted mean, range, variance, standard deviation) and understand the distinction between a statistic and a parameter.

Example: Use spreadsheet formulas to compute measures that summarize your dropout data by state.

IM3.3.5 Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term *variable*.

Example: Compare the data displayed in various forms in the first example. What do you notice about the impact the type of display has on the analysis of the data?

IM3.3.6 Use simulations to construct empirical probability distributions.

Example: Describe how you could use two coins to set up a simulation of a random phenomenon that has a 25 percent chance of a desired outcome.

IM3.3.7 Know and apply the characteristics of the normal distribution.

- Identify settings in which the normal distribution may be useful.
- Determine whether a set of data appears to be uniform, skewed or normally distributed.
- Use the empirical rule to find probabilities that an event will occur in a specific interval that can be described in terms of one, two or three standard deviations from the mean.

  Example: Math SAT scores are normally distributed with mean 500 and standard deviation 100. What is the probability that a randomly selected student's SAT score is greater than 600?

IM3.3.8 Compute and interpret the expected value of random variables in simple cases.

Example: When you flip a coin five times, the number of heads is 0, 1, 2, 3, 4 or 5. Find the probability of each number of heads and draw a histogram of the results.

IM3.3.9 Compute the probability of compound events.

Example: An experiment consists of rolling a die three times and noting the number that lands on top at each throw. Find the probability of a 6, followed by any even number, followed by a 4.

IM3.3.10 Model and solve problems, including probability problems, using counting techniques.

Example: You know that your locker combination contains the numbers 2, 4, 6 and 8, but you have forgotten the order in which they occur. What is the probability that your locker opens with the first combination you try?

Standard 4
Discrete Mathematics

IM3.4.1 Know and use the concepts of sets, elements and subsets.

Example: In the set of integers {…,-3, -2, -1, 0, 1, 2, 3…}, write the subset of even integers.

IM3.4.2 Perform operations on sets (union, intersection, complement, cross product).

Example: In the set of integers from 1 to 10, a subset is the prime numbers. Name the elements in this subset's complement.

IM3.4.3 Identify and give examples of undefined terms, axioms and theorems.

Example: Do you prove axioms from theorems or theorems from axioms?

IM3.4.4 Describe logical statements using the terms *assumption*, *hypothesis*, *conclusion*, *converse*, *inverse* and *contrapositive*. Find the converse, inverse and contrapositive of statements.

Example: Find the converse of "If today is Sunday, then we have school tomorrow." Is the converse logically equivalent to the original statement? Explain.

IM3.4.5 Explain and illustrate the role of definitions, conjectures, theorems, proofs and counterexamples in mathematical reasoning. Construct logical arguments, assess the validity of logical arguments and give counterexamples to disprove statements.

Example: Find an example to show that triangles with two sides and one angle equal are not necessarily congruent.

IM3.4.6 Model and solve problems involving patterns using recursion and iteration, growth and decay, and compound interest.

Example: How many handshakes would occur in this room if everyone shook hands with everyone else? Create a spreadsheet that will find the number of handshakes starting with one person and increasing the number to 50.

IM3.4.7 Use mathematical induction to prove simple propositions.

Example: Use mathematical induction to prove the sum of the first n even positive integers is $n(n+1)$.

IM3.4.8 Use graph-coloring techniques to solve problems.

Example: Color a map of the Midwestern states of the United States so that no adjacent states are the same color. What is the minimum number of colors needed?

IM3.4.9 Use bin-packing techniques to solve problems of optimizing resource usage.

Example: Six large crates of electronic equipment are to be shipped to a warehouse. The crates weigh 2,800, 6,000, 5,400, 1,600, 6,800 and 5,000 pounds. Each delivery truck has a capacity of 10,000 pounds. What is the minimum number of trucks needed to send all the crates?

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.

- Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
- The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or not: Creating a high school diploma that counts.* Washington, DC: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston VA: author.